7.5 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 17, and 71

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 43, 44, 64, 71, and 73

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies available for all exercises

Basic:
Day 1: pp. 510–513
Exs. 1–11, 15–20, 31–36, 43–48, 69–72, 75, 78, 84

Average:
Day 1: pp. 510–513
Exs. 1, 2, 10–12, 21–26, 31, 32, 37–40, 43, 44, 49–54, 61–64, 69–73, 76, 79, 85, 89

Advanced:
Day 1: pp. 510–513
Exs. 1, 2, 12–14, 26–30, 39–44, 53–68*, 71–74*, 77, 80, 91

Block:
pp. 510–513
Exs. 1, 2, 10–12, 21–26, 31, 32, 37–40, 43, 44, 49–54, 61–64, 69–73, 76, 79, 85, 89 (with 7.4)

Differentiated Instruction

See *Algebra 2 Best Practices Toolkit* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 8, 18, 34, 46, 69

Average: 10, 22, 38, 52, 70

Advanced: 12, 28, 42, 54, 71

Extra Practice

- Student Edition, p. 1016
- Chapter 7 Resource Book: Practice levels A, B, C, pp. 54–56

Practice Worksheet

An easily-readable reduced practice page (with answers) for this lesson can be found on p. 476C.

2. Use common logarithms in the change of base formula, or use natural logarithms in the change of base formula.

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: To condense the expression $\log_3 2x + \log_3 y$, you need to use the ? property of logarithms. product

2. ★ **WRITING** *Describe* two ways to evaluate $\log_7 12$ using a calculator. See margin.

EXAMPLE 1 on p. 507 for Exs. 3–14

MATCHING EXPRESSIONS **Match the expression with the logarithm that has the same value.**

3. $\ln 6 - \ln 2$ B
4. $2 \ln 6$ D
5. $6 \ln 2$ A
6. $\ln 6 + \ln 2$ C

A. $\ln 64$ B. $\ln 3$ C. $\ln 12$ D. $\ln 36$

APPROXIMATING EXPRESSIONS **Use $\log 4 \approx 0.602$ and $\log 12 \approx 1.079$ to evaluate the logarithm.**

7. $\log 3$ 0.477
8. $\log 48$ 1.681
9. $\log 16$ 1.204
10. $\log 64$ 1.806
11. $\log 144$ 2.158
12. $\log \frac{1}{3}$ −0.477
13. $\log \frac{1}{4}$ −0.602
14. $\log \frac{1}{12}$ −1.079

21. $\log_4 x - (\log_4 3 + \log_4 y)$

EXAMPLE 2 on p. 508 for Exs. 15–32

EXPANDING EXPRESSIONS **Expand the expression.**

15. $\log_3 4x$ $\log_3 4 + \log_3 x$
16. $\ln 15x$ $\ln 15 + \ln x$
17. $\log 3x^4$ $\log 3 + 4 \log x$
18. $\log_5 x^5$ $5 \log_5 x$
19. $\log_2 \frac{2}{5}$ $\log_2 2 - \log_2 5$
20. $\ln \frac{12}{5}$ $\ln 12 - \ln 5$
21. $\log_4 \frac{x}{3y}$ $\log_4 x - (\log_4 3 + \log_4 y)$
22. $\ln 4x^2y$ $\ln 4 + 2 \ln x + \ln y$
23. $\log_7 5x^3yz^2$ $\log_7 5 + 3\log_7 x + \log_7 y + 2\log_7 z$
24. $\log_6 36x^2$ $\log_6 36 + 2 \log_6 x$
25. $\ln x^2y^{1/3}$ $2 \ln x + \frac{1}{3} \ln y$
26. $\log 10x^3$ $\log 10 + 3 \log x$
27. $\log_2 \sqrt{x}$ $\frac{1}{2} \log_2 x$
28. $\ln \frac{6x^2}{y^4}$ $\ln 6 + 2 \ln x - 4 \ln y$
29. $\ln \sqrt[4]{x^3}$ $\frac{3}{4} \ln x$
30. $\log_3 \sqrt{9x}$ $\frac{1}{2}(\log_3 9 + \log_3 x)$

31. The two parts should be added, not multiplied; $\log_2 5 + \log_2 x$.

32. The power of 3 is attached to the x, not the 8; $\ln 8 + 3 \ln x$.

ERROR ANALYSIS ***Describe*** **and correct the error in expanding the logarithmic expression.**

31. $\log_2 5x = (\log_2 5)(\log_2 x)$ ✗

32. $\ln 8x^3 = 3 \ln 8 + \ln x$ ✗

EXAMPLE 3 on p. 508 for Exs. 33–43

CONDENSING EXPRESSIONS **Condense the expression.**

33. $\log_4 7 - \log_4 10$ $\log_4 \frac{7}{10}$
34. $\ln 12 - \ln 4$ $\ln 3$
35. $2 \log x + \log 11$ $\log 11x^2$
36. $6 \ln x + 4 \ln y$ $\ln x^6y^4$
37. $5 \log x - 4 \log y$ $\log \frac{x^5}{y^4}$
38. $5 \log_4 2 + 7 \log_4 x + 4 \log_4 y$ $\log_4 32x^7y^4$
39. $\ln 40 + 2 \ln \frac{1}{2} + \ln x$ $\ln 10x$
40. $\log_5 4 + \frac{1}{3} \log_5 x$ $\log_3 4 \sqrt[3]{x}$
41. $6 \ln 2 - 4 \ln y$ $\ln \frac{64}{y^4}$
42. $2(\log_3 20 - \log_3 4) + 0.5 \log_3 4$ $\log_3 50$

43. ★ **MULTIPLE CHOICE** Which of the following is equivalent to $3 \log_4 6$? C

(A) $\log_4 18$ (B) $\log_4 72$ (C) $\log_4 216$ (D) $\log_4 256$

44. ★ **MULTIPLE CHOICE** Which of the following statements is *not* correct? **D**

Ⓐ $\log_3 48 = \log_3 16 + \log_3 3$ Ⓑ $\log_3 48 = 3\log_3 2 + \log_3 6$

Ⓒ $\log_3 48 = 2\log_3 4 + \log_3 3$ Ⓓ $\log_3 48 = \log_3 8 + 2\log_3 3$

EXAMPLE 4 B on p. 509 for Exs. 45–61

CHANGE-OF-BASE FORMULA **Use the change-of-base formula to evaluate the logarithm.**

45. $\log_4 7$ **about 1.404** **46.** $\log_5 13$ **about 1.594** **47.** $\log_3 15$ **about 2.465** **48.** $\log_8 22$ **about 1.486**

49. $\log_3 6$ **about 1.631** **50.** $\log_5 14$ **about 1.640** **51.** $\log_6 17$ **about 1.581** **52.** $\log_2 28$ **about 4.807**

53. $\log_7 19$ **about 1.513** **54.** $\log_4 48$ **about 2.792** **55.** $\log_9 27$ **1.5** **56.** $\log_8 32$ **about 1.667**

57. $\log_6 \frac{24}{5}$ **about 0.875** **58.** $\log_2 \frac{15}{7}$ **about 1.100** **59.** $\log_3 \frac{9}{40}$ **about −1.358** **60.** $\log_7 \frac{3}{16}$ **about −0.860**

61. **ERROR ANALYSIS** *Describe* and correct the error in using the change-of-base formula.

$\log_3 7 = \frac{\log 3}{\log 7}$ ✗

When using the change of base formula, the base goes in the denominator; $\frac{\log 7}{\log 3}$.

EXAMPLE 5 on p. 509 for Exs. 62–63

SOUND INTENSITY **In Exercises 62 and 63, use the function in Example 5.**

62. Find the decibel level of the sound made by each object shown below.

a.

Barking dog: $I = 10^{-4}$ W/m²
80 decibels

b.

Ambulance siren: $I = 10^{0}$ W/m²
120 decibels

c.

Bee: $I = 10^{-6.5}$ W/m²
55 decibels

63. The intensity of the sound of a trumpet is 10^3 watts per square meter. Find the decibel level of a trumpet. **150 decibels**

64. ★ **OPEN-ENDED MATH** For each statement, find positive numbers M, N, and b (with $b \neq 1$) that show the statement is false in general.

a–b. Sample answers are given.

a. $\log_b (M + N) = \log_b M + \log_b N$
$M = 5$, $N = 6$, $b = 3$, $2.183 \neq 1.465 + 1.631$

b. $\log_b (M - N) = \log_b M - \log_b N$
$M = 6$, $N = 5$, $b = 3$, $0 \neq 1.631 - 1.465$

C **CHALLENGE** **In Exercises 65–68, use the given hint and properties of exponents to prove the property of logarithms.** **65–68. See margin.**

65. **Product property** $\log_b mn = \log_b m + \log_b n$
(*Hint:* Let $x = \log_b m$ and let $y = \log_b n$. Then $m = b^x$ and $n = b^y$.)

66. **Quotient property** $\log_b \frac{m}{n} = \log_b m - \log_b n$
(*Hint:* Let $x = \log_b m$ and let $y = \log_b n$. Then $m = b^x$ and $n = b^y$.)

67. **Power property** $\log_b m^n = n \log_b m$
(*Hint:* Let $x = \log_b m$. Then $m = b^x$ and $m^n = b^{nx}$.)

68. **Change-of-base formula** $\log_c a = \frac{\log_b a}{\log_b c}$
(*Hint:* Let $x = \log_b a$, $y = \log_b c$, and $z = \log_c a$. Then $a = b^x$, $c = b^y$, and $a = c^z$, so that $b^x = c^z$.)

Avoiding Common Errors

Exercise 21 Some students may write the answer as $\log_4 x - \log_4 3 + \log_4 y$, rather than $\log_4 x - \log_4 3 - \log_4 y$ because they see the product $3y$. Show these students how to expand the given logarithmic expression step-by-step so that the denominator is first treated as a unit: $\log_4 \frac{x}{3y} = \log_4 x - \log_4 3y$ $= \log_4 x - (\log_4 3 + \log_4 y)$ $= \log_4 x - \log_4 3 - \log_4 y$.

Study Strategy

Exercises 45–60 Students may have trouble remembering whether $\log_4 7$ is equal to $\frac{\log 7}{\log 4}$ or $\frac{\log 4}{\log 7}$, and similarly in the other exercises in this group. These students may find it easier to remember that the base in the logarithmic expression is in the denominator (as the "base") of the fraction.

65. Let $x = \log_b m$ and $y = \log_b n$, convert these to exponential form to get $m = b^x$ and $n = b^y$. Then $mn = b^x b^y$; $mn = b^{x+y}$. Convert this to logarithmic form to get $\log_b (mn) = x + y$. Using substitution, to get $\log_b (mn) = \log_b m + \log_b n$.

66. Let $x = \log_b m$ and $y = \log_b n$. Convert these to exponential form and you get $m = b^x$ and $n = b^y$. Then $\frac{m}{n} = \frac{b^x}{b^y}$ which simplifies to b^{x-y}. Convert this to logarithmic form and you get $\log_b \left(\frac{m}{n}\right) = x - y$. Use substitution to get $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$.

67. Let $x = \log_b m$, convert this to exponential form, and you have $m = b^x$ and then $m^n = b^{nx}$. When you convert this to logarithmic form, you have $\log_b m^n = nx$. Use substitution to get $\log_b m^n = n \log_b m$.

68. Let $x = \log_b a$, $y = \log_b c$, and $z = \log_c a$. Convert these to exponential form to get $a = b^x$, $c = b^y$, $a = c^z$. Since $a = b^x$ and $a = c^z$, then $b^x = c^z$. Then take the log of both sides, to end up with $\log_b b^x = \log_b c^z$. This simplifies to $x \log_b b = z \log_b c$. Since $\log_b b = 1$ and $\log_b c = y$, use substitution to get $x = yz$. Then solve for z and the result is $z = \frac{x}{y}$, and by substitution, $\log_c a = \frac{\log_b a}{\log_b c}$.